**Pre-Calculus 2-5 Rational Functions Investigations Name:**

In this section, we will be exploring the *asymptotes* of rational functions.

1. What do you notice about the graph of a function when we have real zeros in the denominator?

For example:  , 

2. In a rational function, what pattern exists when the degree of the polynomial in the numerator ***is less than*** the degree of the polynomial in the denominator? Come up with a conjecture that you can support with three examples.

3. In a rational function, what happens when the degree of the polynomial in the numerator and denominator are the same? Explore the following examples. What is the horizontal asymptote of each? What in the original function could tell you this without graphing? Make a conjecture.

a.  b.  c. 

4. In a rational function, what horizontal asymptote exists when the degree of the polynomial in the numerator ***is greater than*** the degree of the polynomial in the denominator? Come up with a conjecture that you can support with three examples.

5. Let’s consider a rational function where the degree of the numerator is one greater than the degree of the denominator. Another type of asymptote exists for this case. It is called an oblique asymptote.

Graph the function h(x) = .

This function has a vertical asymptote at the real zero in the denominator and an oblique asymptote. Divide out the denominator. Graph the resulting function, *ignoring the remainder*. How does this line relate to the original function?

Let’s try this again. Graph the function g(x) = .

This function has a vertical asymptote at the real zero in the denominator and an oblique asymptote. Divide out the denominator. Graph the resulting function, *ignoring the remainder*. How does this line relate to the original graph of the function?

**Exploring Domain of a Rational Function**

Consider the function: f(x) = . What is the domain of this function?

Factor the numerator and simplify the function. Graph the result. What type of discontinuity does this graph have and where?

**Conclusion**

a. Given a function, how do we know if there will be a vertical asymptote?

b. What has to be true about the degrees of the numerator and denominator of a rational function for the following cases?

-We get a horizontal asymptote at y=0?

-We get a horizontal asymptote that is a fraction of the leading coefficients of the numerator and denominator?

-There is no horizontal asymptote?

c. What happens on the graph when we have a rational function with a numerator and denominator that share a binomial factor?